241. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

If $a^2 + b^2 = c^2$, where a, b, and c are integers, then prove that abc will be a multiple of 60.

In the next issue we shall reprint all unsolved problems in Number Theory published since January, 1913. They are numbers 191, 192, 196, 198, 201, 202, 205, 208, 209, 211, 214, 217, 219, 221, 222, 223. Please have these in mind. EDITORS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

433. Proposed by B. J. BROWN, Student at Drury College.

Prove that, if all the quantities, a, b, etc., are real, then all the roots of the equati ns

$$\begin{vmatrix} a-x & h \\ h & b-x \end{vmatrix} = 0, \qquad \begin{vmatrix} a-x & h & g \\ h & l-x & f \\ g & f & c-x \end{vmatrix} = 0$$

are real; and generalize the proposition.

SOLUTION BY WM. E. HEAL, Washington, D. C.

The first equation may be written

$$\left[x - \left(\frac{a+b}{2}\right)\right]^2 = \frac{(a-b)^2}{4} + h^2.$$

Since the second member is the sum of two squares and so can never become negative, if a, b, and h are real, it follows that both roots are real.

The second equation is proved, in Salmon's *Modern Higher Algebra*, 4th edition, page 28, to have its roots all real.

The general equation, of which the above are special cases, is shown on page 48 of the same work to have all its roots real.

Thus also referred to by A. M. HARDING.

442. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Show that the sum of n terms of the series $1/2 - 1/3 + 1/4 - 1/6 + 1/8 - 1/12 + \cdots$ is $1/3[1 - (1/2)^{n/2}]$ when n is even, and $1/3[1 + 2\sqrt{2}(1/2)^{(n/2)+1}]$ when n is odd.

SOLUTION BY IRBY C. NICHOLS, Chicago, Ill.

(1) *When n is even.* Grouping the terms successively by twos, we have a series of $n/2$ terms from which the factor $(\frac{1}{2} - \frac{1}{3})$ may be removed, thus,

$$(\tfrac{1}{2} - \tfrac{1}{3})\left[\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \cdots \frac{1}{2^{n/2-1}}\right].$$

The series in brackets is geometrical, and we have the sum

$$(\tfrac{1}{2} - \tfrac{1}{3})\left[\frac{1 - \frac{1}{2}\cdot\frac{1}{2^{n/2-1}}}{\frac{1}{2}}\right] = \tfrac{1}{3}[1 - (\tfrac{1}{2})^{n/2}].$$

(2) *When n is odd.* Then the sum of $n+1$ terms can be written by (1), using $n+1$ for n. If now we add the $(n+1)$th term to this sum, we shall have the sum of n terms, since the $(n+1)$th term is negative. This gives

$$\begin{aligned} S_n &= \tfrac{1}{3}[1 - (\tfrac{1}{2})^{(n+1)/2}] + (\tfrac{1}{2})^{[(n+1)/2]-1} \\ &= \tfrac{1}{3}[1 - 2^{1/2}(\tfrac{1}{2})^{n/2+1}] + 2^{3/2}(\tfrac{1}{2})^{n/2+1} \\ &= \tfrac{1}{3}[1 + \sqrt{2}(\tfrac{1}{2})^{n/2+1}]. \end{aligned}$$

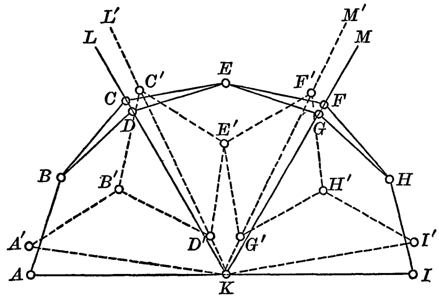
The last result was incorrectly stated by the Proposer.

Also solved by Herbert N. Carleton, J. A. Caparo, F. L. Griffin, Frank R. Morris, G. W. Hartwell, C. E. Horne, H. C. Feemster, Horace Olson, J. V. Balch, H. L. Agard, and the Proposer.

GEOMETRY.

427. Proposed by F. CAJORI, Colorado College.

In S. Gross's linkage for trisection of angles, shown in the figure (KL' and KM' being the trisectors of $A'KI'$), C is fixed on KL, also F on KM; at starting, C and D coincide, also F and G; D slides on KL, G slides simultaneously on KM; if E moves along a perpendicular to AI erected at K, find the loci of B and D.



Solution by J. W. Clawson, Collegeville, Pa.

Denote length of KC by b, $EC = CB = BA$ by a. Now $KA = b$, and, projecting the broken line $KCBA$ on KA,

$$b \cos \pi/3 + a \cos \pi/4 + a \cos 5\pi/12 = b.$$

Hence

$$b = 2a \cos \pi/12.$$

Denote length of KD' at any time by r, angle EKD' by θ, angle $E'C'K = E'D'C'$ by ψ. Then $r + 2a \cos \psi = b$. But, by the law of sines,

$$\frac{\sin (\theta + \psi)}{\sin \theta} = \frac{b}{a}.$$

Solving for $\cos \psi$,

$$\cos \psi = \frac{b}{a} \sin^2 \theta \pm \frac{\cos \theta}{a} \sqrt{a^2 - b^2 \sin^2 \theta}.$$

Hence,

$$r - b + 2b \sin^2 \theta = \mp 2 \cos \theta \sqrt{a^2 - b^2 \sin^2 \theta},$$

i. e.,

$$(r - b \cos 2\theta)^2 = 4 \cos^2 \theta (a^2 - b^2 \sin^2 \theta).$$

This is the polar (r, θ) equation of the locus of D' with respect to KE as initial line and K as pole.

Again, by the law of cosines, $KB'^2 = b^2 + a^2 - 2ab \cos \psi$. Denote length of KB' by ρ and $\angle EKB'$ by ϕ. The equation just written becomes

$$(\rho^2 - a^2 - b^2 \cos \phi)^2 = 2b^2(1 + \cos \phi) \left\{ a^2 - b^2 \frac{1 - \cos \phi}{2} \right\}.$$

This is the polar (ρ, ϕ) equation of the locus of B' where KE is initial line and K is pole.

463. Proposed by NATHAN ALTSHILLER, University of Washington, Seattle.

Through a given point, to draw a line that cuts off on the sides of a given angle two segments the sum of which has a given value.